The time value of money

Present value

Future value

Present Value – Looking backwards

How much money do I need to invest now to have a final value of €20000 in 1 year's time given an AER of 3%?

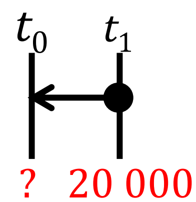
What is the **present value** of a future payment of €20000 in 1 year's time assuming an AER of 3%?

$$20000 = P(1.03)$$

$$\frac{20000}{1.03} = P$$

$$€19417.48 = P$$

$$F = P(1+i)^t$$



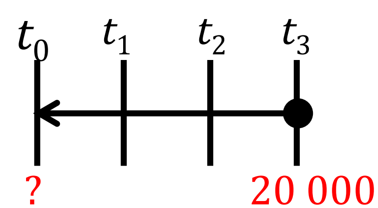
What is the present value of a future payment of €20000 in 3 years' time assuming an AER of 3%?

$$20000 = P(1.03)^3$$

$$\frac{20000}{(1.03)^3} = P$$

$$€18\,302.83 = P$$

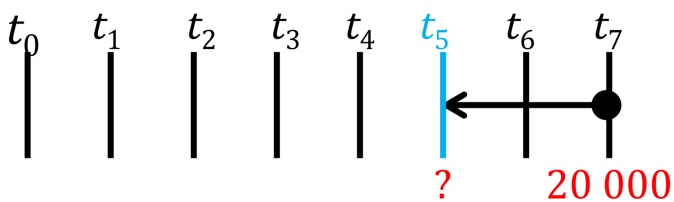
$$F = P(1+i)^t$$



Example

I am assuming that, in the long run, money can be invested at an inflation adjusted annual rate of 3% AER. I am due to be paid €20,000 seven years from now. I wish to collect this debt early, at the end of 5 years from now.

How much should I expect to collect?



$$P = \frac{F}{(1+i)^t}$$

Present value at t_0 : $$P_0 = \frac{20\,000}{1.03^7} = €16261.83$$

$$F = P(1+i)^t$$

Final value at t_5 : $$F_5 = €16261.83(1.03)^5 = €18851.92$$

or $$F = \frac{20000(1.03)^5}{1.03^7}$$

Present Value

Present Value is the value **on a given date** <u>of a future payment</u> (or series of future payments) discounted to reflect the **time value of money** (and other factors such as investment risk).

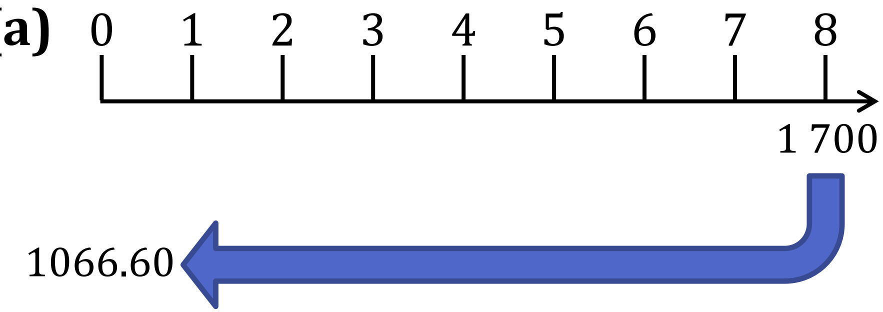
Present Value

Patrick borrows a sum of money from Angela. He promises to pay Angela €1700 in 8 years' time to settle the debt and gives her a written statement to this effect.

(a) Assuming a rate of 6%, paid and compounded annually, how much has he borrowed from Angela?

(b) In 5 years' time how much will he owe Angela?

(a)



$$P = \frac{F}{(1+i)^t}$$

$$= \frac{1700}{(1+0.06)^8}$$

$$= €1066.60$$

(b)

$$F = P(1+i)^t$$

$$= \frac{1700}{(1+0.06)^8}(1+0.06)^5$$

$$= \frac{1700}{(1+0.06)^3}$$

$$= €1427$$

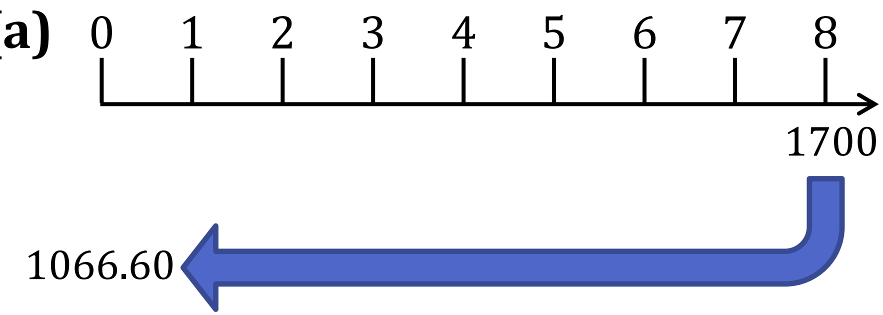
Bonds & Present Value

Angela invested in a savings bond which will be worth €1700 (i.e. future value) in 8 years' time.

(a) Assuming a rate of 6%, paid and compounded annually, how much is the bond worth now?
(What is the present value of the bond? What is the fair market value of the bond now?)

(b) In 5 years' time how much would the bond be worth to Angela?

(a)



$$P = \frac{F}{(1+i)^t}$$

$$= \frac{1700}{(1+0.06)^8}$$

$$= €1066.60$$

(b)

$$F = P(1+i)^t$$

$$= \frac{1700}{(1+0.06)^8}(1+0.06)^5$$

$$= \frac{1700}{(1+0.06)^3}$$

$$= €1427$$

Bonds

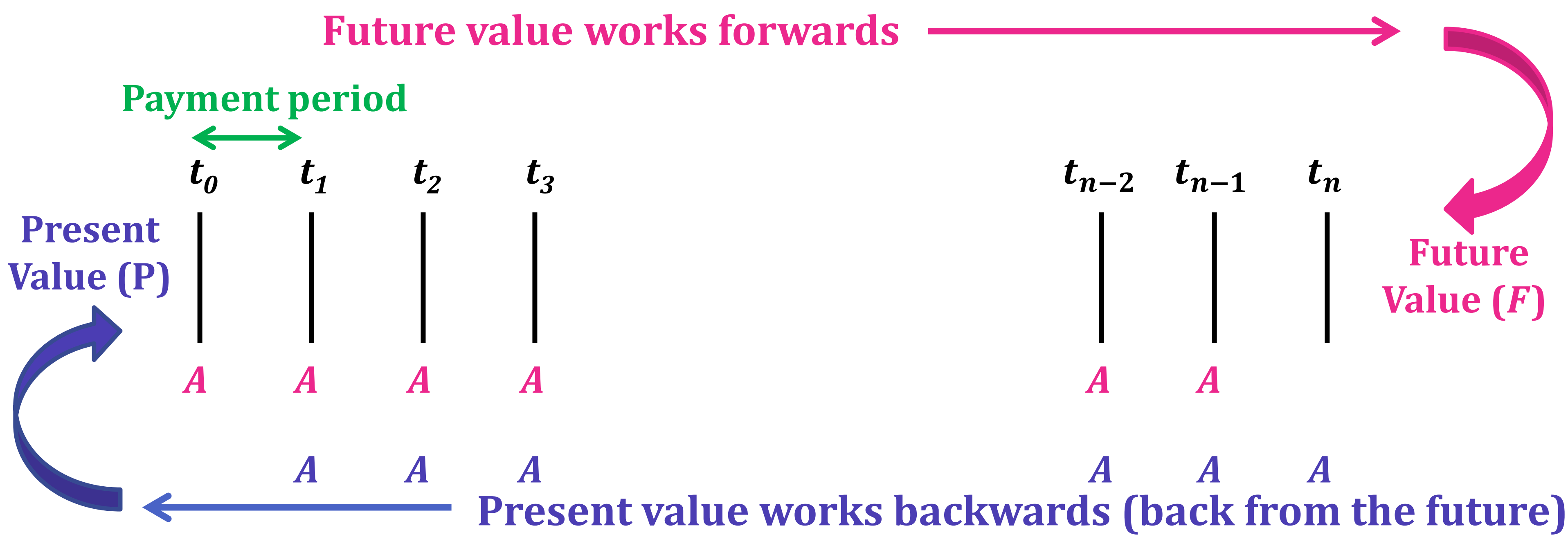
Bonds: A bond is a certificate issued by a government or a public company **promising** to repay borrowed money at a fixed rate of interest at a specified time.

The first bonds issued by the Republic of Ireland

The issuer of the bond is the borrower and the holder of the bond is the lender.

Annuities Involving Present Values

Equal payments of €A each, at equal intervals, for a fixed time



If we are **investing money** into an account, we will be working with the **future values** of the payments into the account, since we are **saving money for one day in the future**. The future value of an annuity is the sum of the future values of each payment. It forms a geometric series.

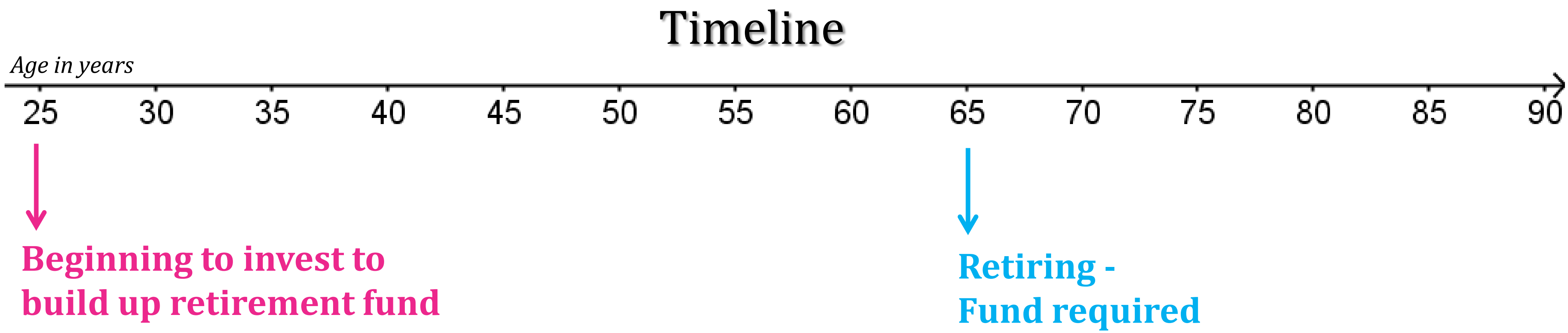
If we are **paying off a loan** then we will be considering the **present value** of a series of payments. This is because **we get the money today,** and pay the money back with interest some time in the future. We also use the idea of present value when **deciding the size of a pension fund**.

Planning a Pension Fund

Annuity involving present value

Question 12 [Sample Paper 1 LCHL Project Schools 2011]

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.



Question 12 [Sample Paper 1 LCHL Project Schools 2011]

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

(a) Write down the present value of a future payment of €20000 in one years' time.

(b) Write down, in terms of *t*, the present value of a future payment of €20000 in *t* years' time.

(a)

$$P = \frac{F}{(1+i)^t}$$

$$P = \frac{20\,000}{(1.03)^1}$$

$$P = €19\,417.48$$

(b)

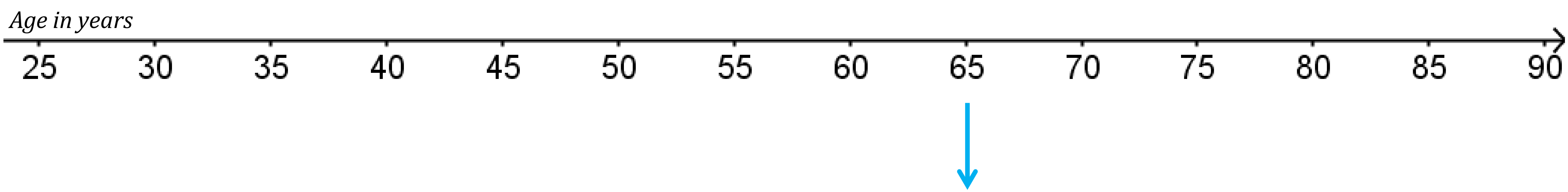
$$P = \frac{20000}{(1.03)^t}$$

Question 12 [Sample Paper 1 LCHL Project Schools 2011]

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value on the date of retirement of the fund required.

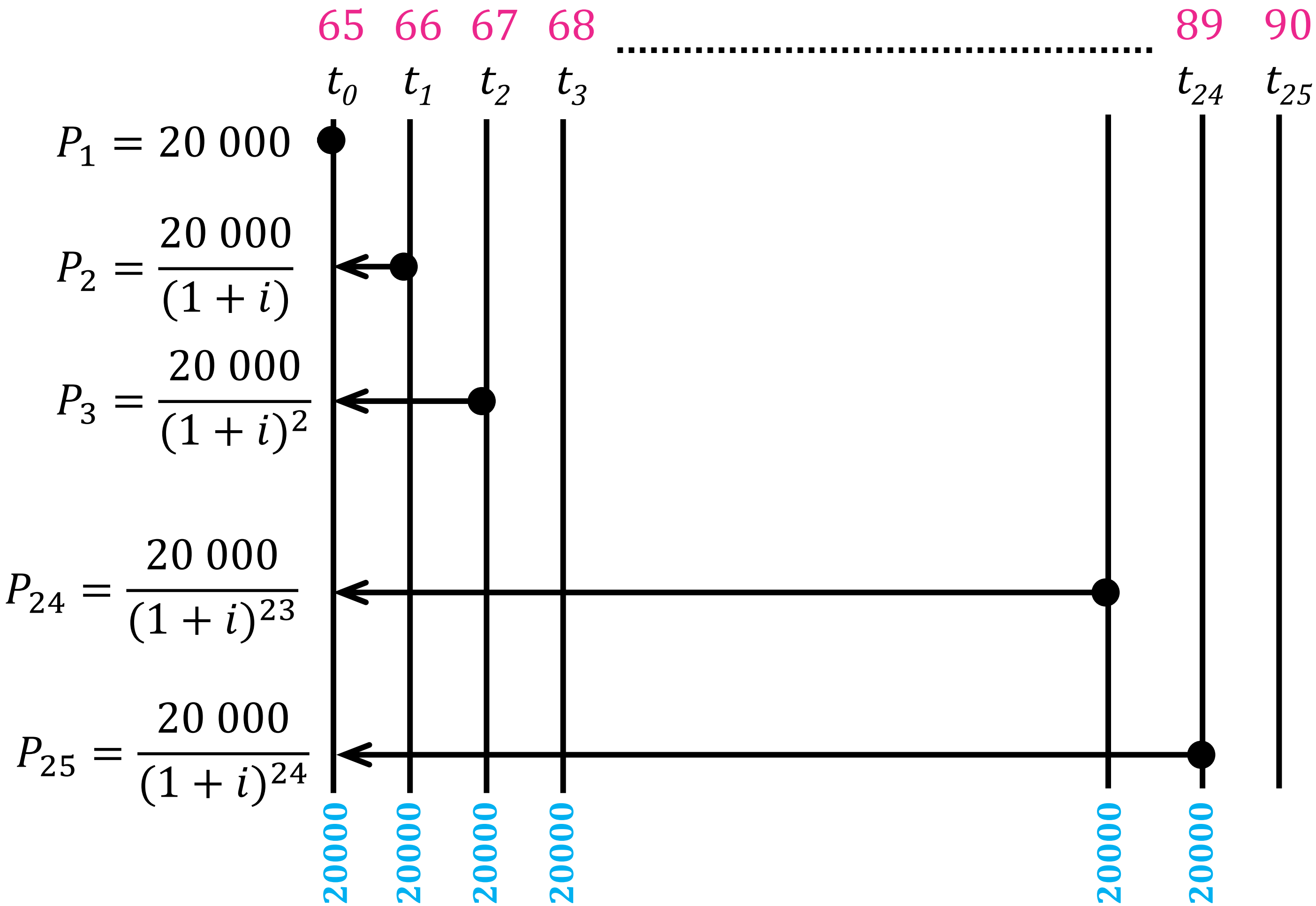
(c)



Will he need 25 × €20 000 in the fund? Explain.

Fund accumulated to give €20,000 per year for 25 years

25 payments of €20 000, each made at the start of each year from the retirement fund.



What is the significance of the sum of all these present values?

It is the money required in the retirement fund on the date of retirement.

Question 12 [Sample Paper 1 LCHL Project Schools 2011]

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value on the date of retirement of the fund required.

(c)

$$S_{25} = \underbrace{20000 + \frac{20000}{(1.03)^1} + \frac{20000}{(1.03)^2} + \cdots + \frac{20000}{(1.03)^{24}}}_{\text{This is a G.P. where } a=20000,\ r=\frac{1}{1.03} \text{ and } n=25}$$

$$S_{25} = \frac{20000\left[1-\left(\frac{1}{1.03}\right)^{25}\right]}{1-\frac{1}{1.03}}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

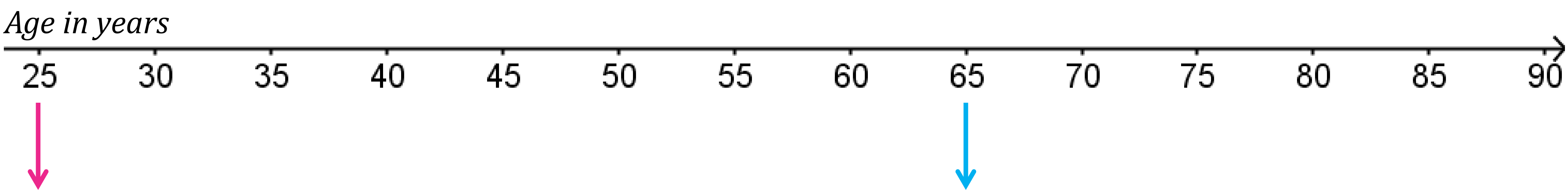
Accumulated fund on the date of retirement = €358710.84

Regular investments to build up the pension fund

Question 12 [Sample Paper 1 LCHL Project Schools 2011]

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**. His retirement is $40 \times 12 = 480$ months away.

(i) Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 3%.



Padraig starts paying regular amounts at the beginning of each month into a pension fund.

A fund of **€358,710.84** must be accumulated by retirement age 65 to give a pension of €20,000 per year for 25 years.

$$F = (1+i)^t$$

$$\Rightarrow 103 = 100(1+i)^{12}$$

$$1.03 = (1+i)^{12}$$

$$\sqrt[12]{1.03} = (1+i)$$

$$1.002466 = 1+i$$

$$0.002466 = i$$

$$0.2466\% = r$$

Question 12 [Sample Paper 1 LCHL Project Schools 2011]

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**. His retirement is $40 \times 12 = 480$ months away.

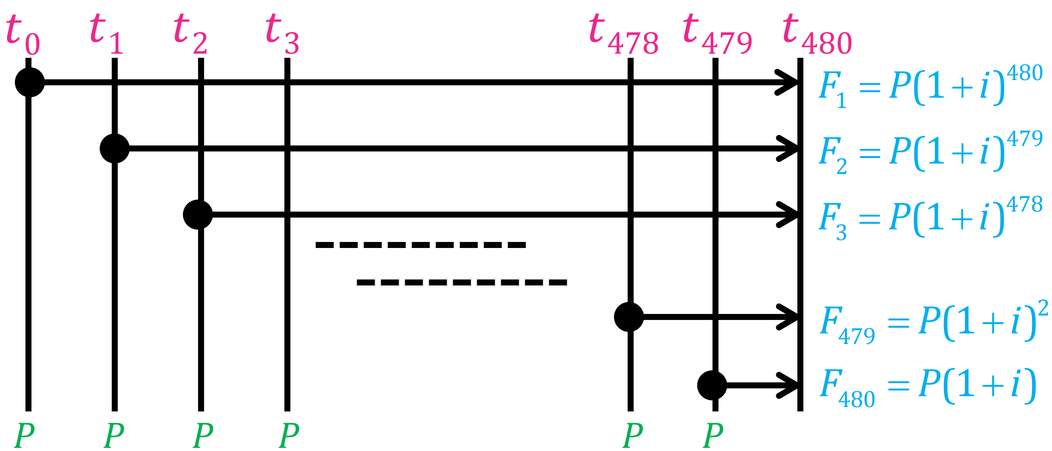
(ii) Write down, in terms of n and P, the value on the retirement date of a payment of €P made n months before the retirement date.

$$F = P(1 + 0.002466)^n$$

Question 12 [Sample Paper 1 LCHL Project Schools 2011]

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**. His retirement is $40 \times 12 = 480$ months away.

(iii) If Pádraig makes 480 equal monthly payments of €P from now until his retirement, what value of P will give the fund he requires?



$$i = \text{the monthly interest rate} = 1.03^{1/12} = 0.002466$$

$$\sum_{r=1}^{n} F_r = \text{The sum of all the separate future values} = \text{The future value of the annuity}$$

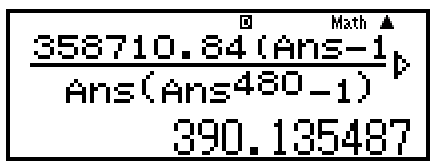
(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**. His retirement is $40 \times 12 = 480$ months away.

(iii) If Pádraig makes 480 equal monthly payments of €P from now until his retirement, what value of P will give the fund he requires?

$$€358{,}710.84 = \underbrace{P\left(1.03^{1/12}\right)^{1} + P\left(1.03^{1/12}\right)^{2} + \cdots + P\left(1.03^{1/12}\right)^{480}}_{a=P(1.03)^{1/12}\quad r=1.03^{1/12}\quad n=480}$$

$$€358{,}710.84 = \frac{P(1.03^{1/12})\left[\left(1.03^{1/12}\right)^{480} - 1\right]}{1.03^{1/12} - 1}$$

$$\frac{358710.84(1.03^{1/12} - 1)}{(1.03^{1/12})\left[\left(1.03^{1/12}\right)^{480} - 1\right]} = P$$



$$€390.17 = P$$

LOANS and APR

Clear Credit Card

13.8% APR

The Clear Credit Card is the low rate Card from Bank of Ireland. It has our low rate of 13.8% APR for purchases* which is one of the most competitive rates around.

Remember you do not have to be an existing Bank of Ireland customer to apply for a Bank of Ireland Clear Credit Card

Apply Online

Features

- Apply online only
- One of the lowest Credit Card APR's available in Ireland
- Add up to three extra cardholders to your account and pay just one annual government stamp duty fee
- Clear balance before due date each month and enjoy up to 56 days of interest-free credit
- 24 Hour Emergency Helpline
- Use your card at 24 million outlets worldwide

* Cash Advance Rate of 26.0% Apr

Lending criteria terms and conditions apply to all credit cards. Credit cards are liable to Government Stamp Duty of €30. Credit cannot be offered to anyone under 18 years of age.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Search
Share
More
Sign In
Email
Password
Login
Add to favourites
Help
wonga.com
home
how it works
about us
my account
Welcome to Wonga.
We can deposit up to £400 in your bank account by 22:35 today.
Existing customers may be able to borrow up to £1,000, depending on your current trust rating.
Instant decision & money sent in 5 minutes » GUARANTEED
how much cash do you want?
265
Max £400
how long do you want it for?
7
Days
(Repayment date: Fri Feb 22 2013)
Borrowing £265 + Interest & fees* £24.18 = Total to repay £289.18
Apply now
Representative example:
Amount of credit £207 for 20 days
Total amount payable £254.42
Interest £41.92
Interest rate 360% pa (fixed)
Transmission fee £5.50
Representative APR 4214%
Representative APR 4214%
*See representative example
APR is not the same as actual interes
VeriSign Secured
APR Explained
98%
while using Wonga
Responsible lending
Wonga is committed to responsible lending and the
https://www.wonga.com/#

$$254.42 = 207(1+i_d)^{20}$$

$$(1+i_d) = 1.010366749$$

$$i_d \approx 0.01 \Rightarrow r_d \approx 1\%$$

$$(1+i_d)^{365} = 43.137$$

$$43.137 = 1 + i \quad \text{where } i = \text{APR expressed as a decimal}$$

$$i = 42.14$$

$$\text{APR} = 4214\%$$

Representative example:

Amount of credit £207 for 20 days
Total amount payable £254.42

Interest £41.92
Interest rate 360% pa (fixed)

Transmission fee £5.50
Representative APR 4214%

All costs must be included when calculating APR

APR (Loans and other forms of credit)

APR (annual percentage rate) is the annual rate of interest payable on mortgages, loans, credit cards etc.

- Used for borrowings
- *Includes all costs* and charges involved in setting up the loan when they apply
- Allows *comparison between loans* with different interest rates and charges
- It takes account of the possible different compounding periods in different products and equalises them all to the equivalent rate compounded **annually.**
- **By law the APR must be quoted in all offers of loans or credit and all advertisements for loans or credit**
- APR is based on the idea of the **present value** of a future payment

Amortisation – Mortgages and other Loans

A **mortgage** is a bank loan specifically designed to purchase a home.
The house functions as the collateral (security pledged for the repayment of the loan).

An **amortized loan** is a loan for which the **loan amount plus interest** is paid off in a series of regular equal payments.
e.g. Term loans and annuity mortgages as opposed to endowment mortgages

The amortized loan **calculates <u>interest on the balance</u> of the loan** after each payment. Hence the interest decreases with each payment.

An **amortisation schedule** is a list of:

- several periods of payments
- the principal and interest portions of those payments
- the outstanding principal (or balance) after each of those payments is made.

Question 14 [Amortisation]

Sean borrows €10,000 at an APR of 6%.
He repays it in five equal instalments of €2373.96 over five years, with the first repayment one year after he takes out the loan.
Fill in the amortisation schedule below.

Payment #	Fixed payment	Interest portion	Principal portion	Balance
0				€10,000.00
1	€ 2373.96	€600.00	€1773.96	€ 8226.04
2	€ 2373.96	€493.56	€1880.40	€6345.53
3	€ 2373.96	€380.74	€1993.22	€4352.41
4	€ 2373.96	€261.14	€2112.82	€2239.59
5	€ 2373.96	€134.38	€2239.59	€0
Totals	**€11869.80**	**€1869.80**	**€10,000.00**	

Rate per year	6.00%
years	5
fixed payment	€2373.96

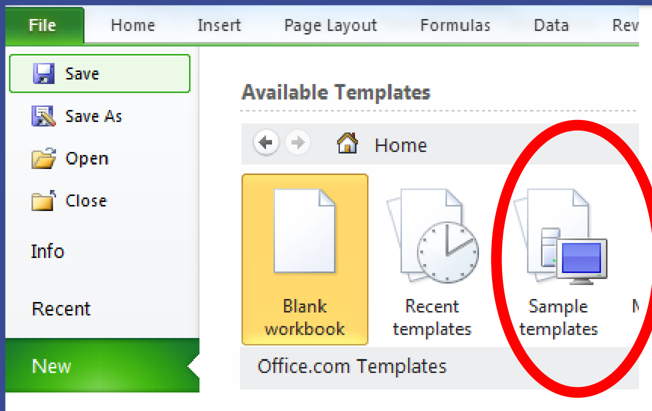
(a) How is the interest portion of each successive payment changing? Why?

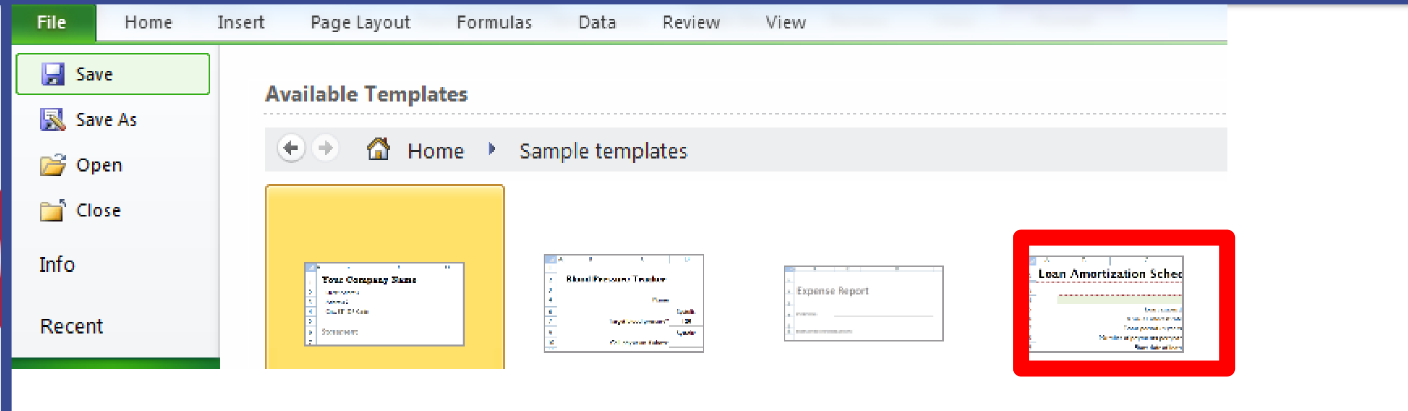
(b) How is the principal portion of each successive payment changing? Why?

(c) What do the principal portions of all the payments add up to?

(d) What do the present values of all the payments add up to?

Loan amortisation Excel template





Loan Amortization Schedule

Enter values	
Loan amount	$ 10,000.00
Annual interest rate	6.00 %
Loan period in years	5
Number of payments per year	1
Start date of loan	01/01/2013
Optional extra payments	

Loan summary	
Scheduled payment	$ 2,373.96
Scheduled number of payments	5
Actual number of payments	5
Total early payments	$ -
Total interest	$ 1,869.82

Lender name:

Pmt. No.	Payment Date	Beginning Balance	Scheduled Payment	Extra Payment	Total Payment	Principal	Interest	Ending Balance	Cumulative Interest
1	01/01/2014	$ 10,000.00	$ 2,373.96	$ -	$ 2,373.96	$ 1,773.96	$ 600.00	$ 8,226.04	$ 600.00
2	01/01/2015	$ 8,226.04	$ 2,373.96	$ -	$ 2,373.96	$ 1,880.40	$ 493.56	$ 6,345.63	$ 1,093.56
3	01/01/2016	$ 6,345.63	$ 2,373.96	$ -	$ 2,373.96	$ 1,993.23	$ 380.74	$ 4,352.41	$ 1,474.30
4	01/01/2017	$ 4,352.41	$ 2,373.96	$ -	$ 2,373.96	$ 2,112.82	$ 261.14	$ 2,239.59	$ 1,735.44
5	01/01/2018	$ 2,239.59	$ 2,373.96	$ -	$ 2,239.59	$ 2,105.21	$ 134.38	$ -	$ 1,869.82

http://www.wolframalpha.com/

WolframAlpha computational knowledge engine

Enter what you want to calculate or know about:

payment schedule

Examples Random

Calculate interest portion of payment

- initial loan amount: 10000 euros
- compounding periods: 5
- interest rate (per period): 6 %
- current period: 1

Assuming interest portion of payment | Use principal portion of payment instead

Assuming initial loan amount | Use payment instead

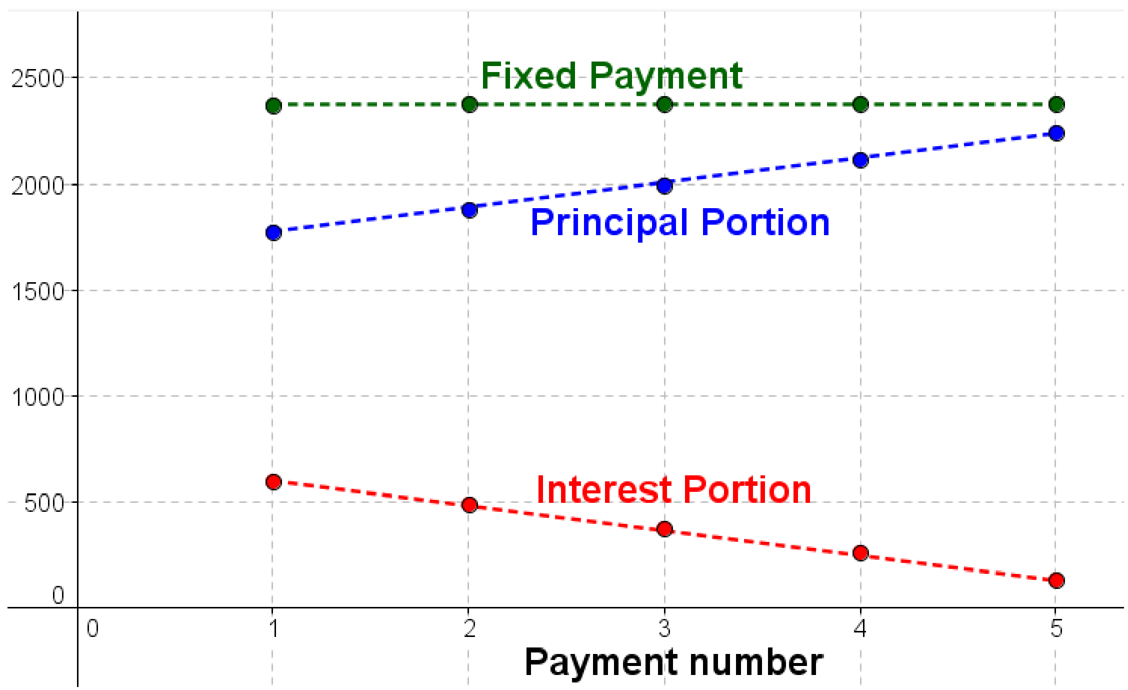
Amortization table:

period	beginning balance	interest	principal	ending balance
1	€10 000	€600	€1773.96	€8226.04
2	€8226.04	€493.56	€1880.40	€6345.63
3	€6345.63	€380.74	€1993.23	€4352.41
4	€4352.41	€261.14	€2112.82	€2239.59
5	€2239.59	€134.38	€2239.59	€0

Amortized loan, 5 years, 5 Annual Payments

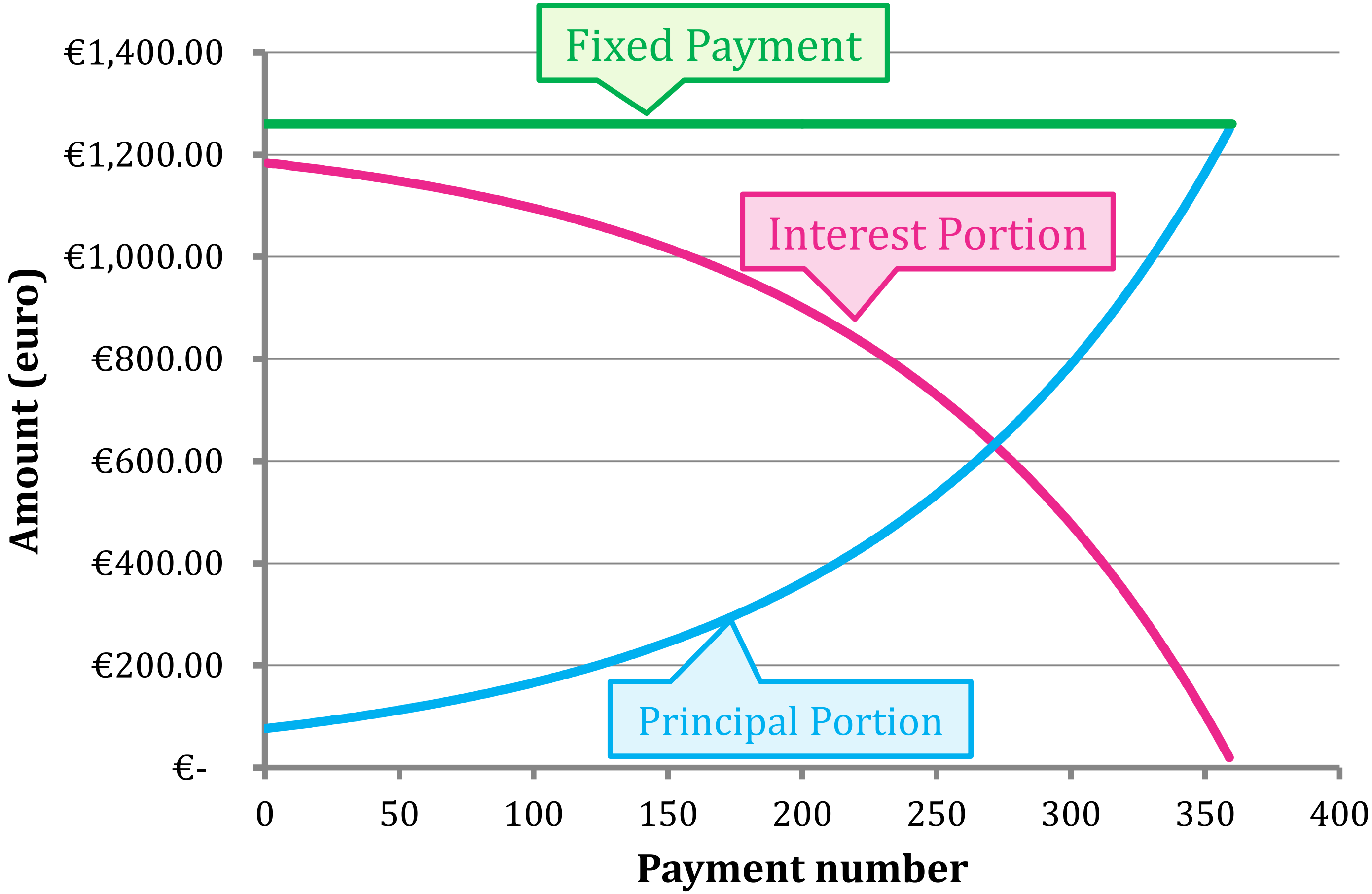
Pymt #	Fixed payment	Interest portion	Principal portion	Balance
0				€ 10,000.00
1	€ 2,373.96	€ 600.00	€ 1,773.96	€ 8,226.04
2	€ 2,373.96	€ 493.56	€ 1,880.40	€ 6,345.63
3	€ 2,373.96	€ 380.74	€ 1,993.23	€ 4,352.41
4	€ 2,373.96	€ 261.14	€ 2,112.82	€ 2,239.59
5	€ 2,373.96	€ 134.38	€ 2,239.59	€ 0.00

Relationship between the interest portion of the payment and the principal portion of the payment with payment number, for an amortized loan (shown graphically)



Amortised Loan, 30 Years, 360 Monthly Payments

Mr. Mooney bought his house in 1975. He obtained a loan from the bank for 30 years at an interest rate of 9.8% APR. His monthly payment at the end of each month was €1260.



The periodic repayment amount remains constant. However as time passes, the interest portion of the repayment reduces and the principal(loan) portion increases.

Question 14 [Amortisation]

Sean borrows €10,000 at an APR of 6%.
He repays it in five equal instalments of €2373.96 over five years, with the first repayment one year after he takes out the loan.

(d) What do the present values of all the payments add up to?

$$P_1 = \frac{2373.96}{(1.06)^1} = €2239.59$$

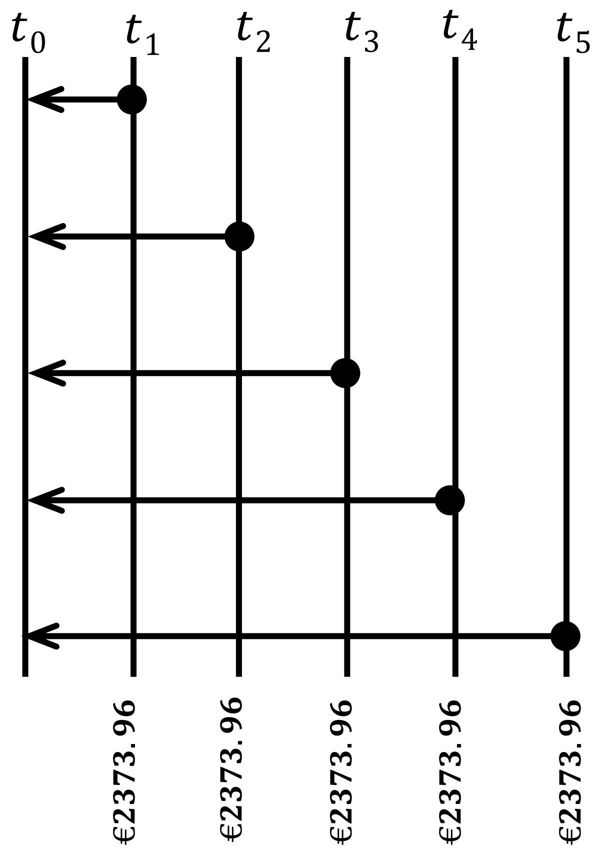
$$P_2 = \frac{2373.96}{(1.06)^2} = €2112.82$$

$$P_3 = \frac{2373.96}{(1.06)^3} = €1993.23$$

$$P_4 = \frac{2373.96}{(1.06)^4} = €1880.40$$

$$P_5 = \frac{2373.96}{(1.06)^5} = €1773.96$$

$$P_1 + P_2 + P_3 + P_4 + P_5 = €10000$$



The sum of all the present values of the repayments A = the sum borrowed (the loan)

The APR is the interest rate for which the present value of all the repayments = the loan amount.

$$P=\underbrace{\frac{A}{(1+i)^1}+\frac{A}{(1+i)^2}+\frac{A}{(1+i)^3}+\frac{A}{(1+i)^4}+\frac{A}{(1+i)^5}}_{a=\frac{A}{1+i},\ r=\frac{1}{1+i},\ n=5}$$

$$P=\frac{\frac{A}{1+i}\left[1-\left(\frac{1}{1+i}\right)^5\right]}{1-\frac{1}{1+i}}$$

$$P=\frac{A\left[1-\frac{1}{(1+i)^5}\right]}{1+i-1}$$

$$P=\frac{A[(1+i)^5-1]}{i(1-i)^5}$$

$$\Rightarrow A=P\frac{i(1+i)^5}{(1+i)^5-1}$$

Check it out!

"Tables and Formulae" page 31

Using the formula to calculate the amount of a fixed mortgage repayment

Sean borrows €10,000 at an APR of 6%. He wants to repay it in five equal instalments over five years, with the first repayment one year after he takes out the loan. How much should each repayment be?

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$= 10000\frac{(0.06)(1+0.06)^5}{(1+0.06)^5 - 1}$$

$$= €2373.96$$

Calculation of the Loan Amount for an Amortised Loan given *A*, *t*, and *i*

Sean takes out a loan at an APR of 6%. He repays it in five instalments of €2373.96 at the end of each year for five years. The first repayment is one year after he takes out the loan. Calculate the loan amount ?

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$P = A\frac{\left[(1+i)^t - 1\right]}{i(1+i)^t}$$

Rearranging the formula

$$= 2373.96\frac{[(1.06)^5 - 1]}{0.06(1.06)^5}$$

$$= €10{,}000$$

A mortgage of €P is taken out and is to be repaid over t years, with equal payments of €A being made at the end of each year.
The APR expressed in decimal form is i.

(i) Show that the sum of the present values of all the repayments is equal to the loan amount

(ii) Derive an expression for the payment amount A, in terms of P, i, and t.

t/years	A/€	Balance outstanding
0		P
1	A	$P(1+i)-A$
2	A	$P(1+i)^2-A(1+i)-A$
3	A	$P(1+i)^3-A(1+i)^2-A(1+i)-A$
4	A	$P(1+i)^4-A(1+i)^3-A(1+i)^2-A(1+i)-A$
t	A	$P(1+i)^t-A(1+i)^{t-2}-A(1+i)^{t-1}-A(1+i)-A$

What is the balance outstanding after t years equal to?

Since the final payment has now been made and the loan is paid off, the balance outstanding at the end of year t is zero.

$$P(1+i)^t - A(1+i)^{t-1} - A(1+i)^{t-2} - \cdots - A(1+i) - A = 0$$

$$\Rightarrow P(1+i)^t = A(1+i)^{t-1} + A(1+i)^{t-2} + \cdots + A(1+i) + A$$

Dividing both sides of the equation by $(1+i)^t$:

$$P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \cdots + \frac{A}{(1+i)^{t-1}} + \frac{A}{(1+i)^t}$$

The loan amount = the sum of the present values of all the repayments

Deriving the Formula for a Mortgage Repayment (Syllabus 3.1 LCHL)

Loan = sum of the present values of the repayments

$$s_n = \frac{a(1-r^n)}{1-r}$$

$$P = \underbrace{\frac{A}{(1+i)^1} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \cdots + \frac{A}{(1+i)^t}}_{a=\frac{A}{1+i},\ r=\frac{1}{1+i},\ n=t}$$

$$P = \frac{\frac{A}{1+i}\left[1-\left(\frac{1}{1+i}\right)^t\right]}{1-\frac{1}{1+i}}$$

$$P = A\frac{\left[1-\frac{1}{(1+i)^t}\right]}{1+i-1}$$

$$P = A\frac{\left[\frac{(1+i)^t-1}{(1+i)^t}\right]}{i}$$

$$\Rightarrow A = P\frac{i(1+i)^t}{(1+i)^t-1}$$

Prepaying a loan

Prepaying a Loan

If Sean wants to pay the loan off after two years how much does he owe?

The outstanding balance on a loan is always calculated directly after the last payment is made.

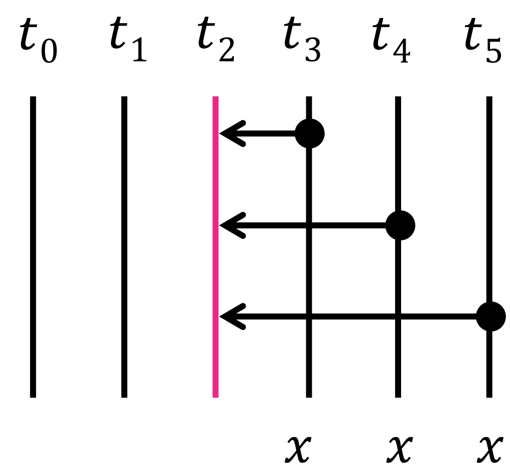
Prepaying a loan of €10,000 borrowed for 5 years at 6 % APR with equal repayments of €2373.96, at the end of two years, (with 3 years to go).

$$P_1 = \frac{2373.96}{(1+0.06)^1} = €2239.59$$

$$P_2 = \frac{2373.96}{(1+0.06)^2} = €2112.82$$

$$P_3 = \frac{2373.96}{(1+0.06)^3} = €1993.23$$

$$P_1 + P_2 + P_3 = €6,345.64$$



The outstanding balance on the loan after 2 years = sum of the present values **at that time** of the 3 remaining repayments.

Prepaying a Loan

If there are 5 repayments, the amount outstanding after the 2nd repayment is the sum of the values on that date (present values) of the remaining (5−2) repayments.

If there are n repayments, the amount outstanding after the r^{th} repayment is the sum of the values on that date (present values) of the remaining $(n - r)$ repayments.

Present values of repayments	P_5	P_4	P_3	P_2	P_1	$\sum_{n=1}^{y} P_n$	Amount paid off the capital sum each year.
Outstanding balance at <u>time 0</u>	$\frac{2373.96}{(1.06)^5}$ = €1773.96	$+\frac{2373.96}{(1.06)^4}$	$+\frac{2373.96}{(1.06)^3}$	$+\frac{2373.96}{(1.06)^2}$	$+\frac{2373.96}{(1.06)^1}$	= €10000	-1773.96
Pay €2373.96 end of yr 1	Outstanding balance at **<u>End of 1 year</u>**	$\frac{2373.96}{(1.06)^4}$ = €1880.40	$+\frac{2373.96}{(1.06)^3}$	$+\frac{2373.96}{(1.06)^2}$	$+\frac{2373.96}{(1.06)^1}$	= €8226.04	-1880.40
	Pay €2373.96 end of yr 2	Outstanding balance at **<u>End of 2 years</u>**	$\frac{2373.96}{(1.06)^3}$ = €1993.23	$+\frac{2373.96}{(1.06)^2}$	$+\frac{2373.96}{(1.06)^1}$	= €6345.63	-1993.23
		Pay €2373.96 end of yr 3	Outstanding balance at **<u>End of 3 years</u>**	$\frac{2373.96}{(1.06)^2}$ = €2112.82	$+\frac{2373.96}{(1.06)^1}$	= €4352.41	-2112.82
			Pay €2373.96 end of yr 4	Outstanding balance at **<u>End of 4 years</u>**	$\frac{2373.96}{(1.06)^1}$ = €2239.58	= €2239.59	-2239.59
				Pay €2373.96 end of yr 5	Outstanding balance at **<u>End of 5 years</u>**	= €0	

Sample Question LCHL Pre-paying a loan (Paying before loan's term is over)

Mr Mooney bought his house on Jan 1st 1975. He obtained a loan from the bank for 30 years at an interest rate of 9.8% APR.

His monthly payment at the end of each month was €1260. In 1995, on Jan 1st Mr Mooney decided to pay off the loan.

Find the balance of the loan i.e. the amount he owed, at that time.

Since this is a question on loan repayments it is a good idea to look at present value(s).

Mr Mooney has made payments for 20 years (240 months) so he still has 120 payments to make.

Hence the bank should charge him the present value of those payments.

Sample Question LCHL Pre-paying a loan (Paying before loan's term is over)

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His monthly payment at the end of each month was €1260. In 1995, on Jan 1st Mr Mooney decided to pay off the loan.
Find the balance of the loan i.e. the amount he owed, at that time.

$x =$ the present value of 120 monthly payments of €1260 due at the end of each month

$$x = \underbrace{\frac{1260}{(1.098^{1/12})^1} + \frac{1260}{(1.098^{1/12})^2} + \cdots + \frac{1260}{(1.098^{1/12})^{119}} + \frac{1260}{(1.098^{1/12})^{120}}}_{a=\frac{1260}{1.098^{1/12}},\ n=120 \text{ and } r=\frac{1}{1.098^{1/12}}}$$

$$= \frac{\dfrac{1260}{1.098^{1/12}}\left[1-\left(\dfrac{1}{1.098^{1/12}}\right)^{120}\right]}{1-\dfrac{1}{1.098^{1/12}}}$$

$$= €97{,}847.55$$